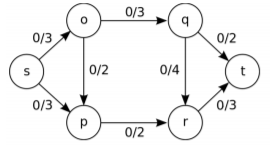
Question 1. Which ones of the following are flows (i.e., they satisfy the definition of Flow given in the lecture)?

**Answer:**

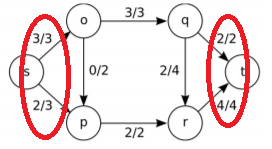
**The flows:**

**图片包含 物体

已生成极高可信度的说明 **

**The not flows:**

图片包含 物体

已生成极高可信度的说明 

**The reason for they are not flows: the left one the f(o,q) is not small and equal to f(q,o), and the right one the sum of source node don not equal to the sum of sink (which are marked with red colour on the picture above).**

Question 2. Consider the following flow network G.

图片包含 天空, 物体, 手表, 照片

已生成极高可信度的说明

And the following flow f:

图片包含 物体, 天空, 照片, 时钟

已生成极高可信度的说明

1. Write down the residual network Gf .

Follow the flow f then:

4

6

6

8

4

4

2

6

10

10

9

1. Identify a path flow in the residual network you found in (1) and Combine it with the flow given above.

The path flow was:

4

6

6

8

4

4

2

6

10

10

9

And the limit weight is 6 in this path (black colour), and then combine it with flow f:

4

10

10

6

6

0

0

0

0

Question 3. Consider the flow network below. Apply Ford-Fulkerson’s algorithm to find a maximal flow and also identify a minimal cut in the network.

5

6

8

9

7

3

3

6

8

4

6

12

9

5

6

0

5

6

7

8

4

**Answer:**

**Iteration 1:**

**Pick:**

**Current Residual Network:**

5

6

8

9

7

3

3

6

8

4

6

12

9

5

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

5

5

5

0

**Update:**

**Current Residual Network:**

5

6

3

9

2

5

3

3

6

5

8

4

6

12

9

5

6

0

5

6

7

8

4

**Iteration 2:**

**Pick:**

**Current Residual Network:**

5

6

3

9

2

5

3

3

6

5

8

4

6

12

9

5

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

2

5

2

7

7

0

**Update:**

**Current Residual Network:**

5

7

8

2

1

7

7

3

3

4

8

4

6

12

9

5

6

0

5

6

7

8

4

**Iteration 3:**

**Pick:**

**Current Residual Network:**

5

7

8

2

1

7

7

3

3

4

8

4

6

12

9

5

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

3

5

2

8

7

1

1

0

**Update:**

**Current Residual Network:**

5

6

8

8

3

7

3

3

3

4

1

5

8

1

12

9

5

6

0

5

6

7

8

4

**Iteration 4:**

**Pick:**

**Current Residual Network:**

5

6

8

8

3

7

3

3

3

4

1

5

8

1

12

9

5

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

6

5

3

2

8

7

4

3

4

0

**Update:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

5

4

2

3

4

12

9

5

6

0

5

6

7

8

4

**Iteration 5:**

**Pick:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

5

4

2

3

4

12

9

5

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

6

5

3

2

8

7

4

5

6

2

2

0

**Update:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

3

4

5

6

10

7

2

7

6

0

5

6

7

8

4

**Iteration 6:**

**Pick:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

3

4

5

6

10

7

2

7

6

0

5

6

7

8

4

**Combine:**

**Current Flow:**

6

5

3

2

8

7

4

8

6

5

5

0

3

3

**Update:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

4

8

6

7

4

10

5

5

6

0

3

3

5

6

4

4

**Iteration 7:**

**Pick:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

4

8

6

7

4

10

5

5

6

0

3

3

5

6

4

4

**Combine:**

**Current Flow:**

6

5

3

2

8

7

4

8

6

5

5

0

4

3

7

4

**Update:**

**Current Residual Network:**

5

3

8

8

6

7

3

3

4

4

8

6

7

4

2

10

1

5

0

4

3

7

5

6

4

4

**After 7 Iterations, we can get**

**Maximal Flow:**

6

5

3

2

8

7

4

8

6

5

5

0

4

3

7

4

**And base on the Residual Network at Iteration 7, the source can reach A, D, C, E, H, F, G, and so the minimal cut will be {(H,t),(E,t),(A,B)} like red colour below:**

**Residual Network:**

5

6

8

9

7

3

3

6

8

4

6

12

9

5

6

0

5

6

7

8

4

Question 4. Consider the following bipartite graph where nodes on the left {1, 2, 3, 4, 5} are being matched to the nodes on the right {6, 7, 8, 9, 10} of an online dating website, and edges represent a profile match. Use flow network to decide on a maximal matching of the graph.

**Answer: Add source and sink to graph and initial capacity of each edge to 1**

1

1

1

1

1

1

1

1

1

1

1

And then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

**Pick a path**, because choose 2 to 6 cannot reach the sink, so pick 2 to 9

1

1

1

1

1

1

1

1

1

1

1

And then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

1

**Pick a path**, because choose 3 to 6 cannot reach the sink, so pick 3 to 7

1

1

1

1

1

1

1

1

1

1

1

1

And then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

1

1

**Pick a path**, because choose 4 to 9 cannot reach the sink, so pick 3 to 8

1

1

1

1

1

1

1

1

1

1

1

1

1

And then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

1

1

1

**To make a Profile match:**

**Pick a path**, because choose 5 to 6 (only path) cannot reach the sink , so to make a profile match, so for 1 need to change path to 8 (the only path for 1, because 1 to 7 which will make 3 only link to 6).so then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

1

1

1

And now the 4 and 1 are linking to 8, so change 4 links to 10 (which don’t affect to 2), so then pick a path for 5.

1

1

1

1

1

1

1

1

1

1

1

1

1

1

And then the Current Residual Network will be

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

So, to make a **profile match (base on the last residual network above)** which so on current flow (the red colour):

1

1

1

1

1

1

1

1

1

1